

1. Simplify each of the following rational expressions.

$$(a) \frac{100x^3y^5}{36xy^8} = \frac{25x^2}{9y^3}$$

$$(c) \frac{-3m^4n}{12m^4n^3} = -\frac{1}{4n^2}$$

$$(b) \frac{48ab^3c^2}{6a^7bc^0} = \frac{8b^2c^2}{a^6}$$

$$(d) \frac{12r^9s^3}{24r^8s^4} = \frac{r}{2s}$$

2. Find the domain of each of the following rational functions. (Remember that dividing by 0 is undefined. Find any value of x that makes the denominator of the function 0; those values of x are *not* in the domain.)

$$(a) f(x) = \frac{1}{x-2}$$

$$\text{dom} = \{x \mid x \neq 2\}$$

$$(c) f(x) = \frac{x}{x^2-4} = \frac{x}{(x-2)(x+2)}$$

$$\text{dom} = \{x \mid x \neq 2, -2\}$$

$$(b) f(x) = \frac{3x-1}{2x-5}$$

$$\text{dom} = \{x \mid x \neq \frac{5}{2}\}$$

$$(d) f(x) = \frac{x}{x^2-4} = \frac{x}{(x-2)(x+2)}$$

$$\text{dom} = \{x \mid x \neq 2, -2\}$$

3. Factor the following polynomials.

$$(a) x^2 - 4 = (x-2)(x+2)$$

$$(d) x^2 + x - 2 = (x+2)(x-1)$$

$$(b) x^2 + 6x + 9 = (x+3)(x+3)$$

$$(e) x^2 + 2x - 3 = (x+3)(x-1)$$

$$(c) x^2 + x - 6 = (x+3)(x-2)$$

4. Use your answers from Question 3 to simplify the following rational expressions.

$$(a) \frac{(x^2+6x+9)(x^2-4)}{(x^2+x-2)(x^2+x-6)} = \frac{\cancel{(x+3)}(x+3)\cancel{(x-2)}(x+2)}{(x+2)\cancel{(x-1)}\cancel{(x+3)}(x-2)} = \frac{x+3}{x-1}$$

$$(b) \frac{(x^2+x-6)(x^2+2x-3)}{(x^2+x-2)(x^2+6x+9)} = \frac{\cancel{(x+3)}(x-2)\cancel{(x+3)}(x+1)}{(x+2)\cancel{(x-1)}\cancel{(x+3)}(x+3)} = \frac{x-2}{x+2}$$

5. State the domain of each of the following rational functions, and simplify.

$$(a) f(x) = \frac{x-5}{x^2-25} = \frac{\cancel{(x-5)}}{(x-5)(x+5)} = \frac{1}{x+5}$$

$$\text{dom} = \{x \mid x \neq -5, 5\}$$

$$(b) f(x) = \frac{x(x-3)^5}{x^3(x-3)^2} = \frac{(x-3)^3}{x^2}$$

$$\text{dom} = \{x \mid x \neq 0, 3\}$$

$$(c) f(y) = \frac{y^2+8y-9}{y^2-5y+4} = \frac{(y+9)\cancel{(y-1)}}{(y-4)\cancel{(y-1)}} = \frac{y+9}{y-4}$$

$$\text{dom} = \{x \mid x \neq 4, 1\}$$

$$(d) f(x) = \frac{x+5}{-x-5} = -\frac{\cancel{(x+5)}}{\cancel{(x+5)}} = -1$$

$$\text{dom} = \{x \mid x \neq -5\}$$

$$(e) f(y) = \frac{y-14}{14-y} = \frac{y-14}{-(y-14)} = -1$$

$$\text{dom} = \{x \mid y \neq 14\}$$

1. Multiply or divide, as indicated. Remember to cancel first!

$$(a) \frac{\cancel{4}^4 w^2}{\cancel{3}_3} \cdot \frac{\cancel{3}}{2w^4} = \frac{4}{3w^2}$$

$$(b) \frac{\cancel{3}^3 \cancel{7}^5}{7s} \cdot \frac{2rs^3}{\cancel{8}^8 r^3 s^2} = \frac{3r^6 s^3}{7r^3 s^3} = \frac{3r^3}{7}$$

$$(c) x(x+5)^2 \cdot \frac{2}{x^2-25} = \frac{2x(x+5)^2}{(x-5)(x+5)} = \frac{2x(x+5)}{(x-5)}$$

$$(d) \frac{6x^2y^2}{(x-2)} \div \frac{3xy^2}{(x-2)^2} = \frac{\cancel{6}^2 \cancel{x}^2 y^2}{(\cancel{x}-2)} \cdot \frac{(x-2)^2}{\cancel{3}^3 x y^2} = \frac{2x(x-2)}{1}$$

$$(e) \frac{t^2+5t}{t+1} \div (t+5) = \frac{t(\cancel{t+5})}{t+1} \cdot \frac{1}{(\cancel{t+5})} = \frac{t}{t+1}$$

$$(f) \frac{2}{25x^2} \cdot \frac{5x}{12} \div \frac{2}{15x} = \frac{\cancel{2}}{25x^2} \cdot \frac{\cancel{5}^5 x}{12} \cdot \frac{\cancel{15}^3 x}{\cancel{2}} = \frac{3x^2}{12x^2} = \frac{1}{4}$$

$$(g) \left(\frac{x^2-4y^2}{x+2y} \div (x+2y) \right) \cdot \frac{2y}{x-2y} = \frac{(\cancel{x-2y})(\cancel{x+2y})}{(x+2y)} \cdot \frac{1}{(\cancel{x+2y})} \cdot \frac{2y}{(\cancel{x-2y})}$$

$$= \frac{2y}{x+2y}$$

2. Add or subtract, as indicated. Start by finding the Least Common Denominator (LCD).
Remember to simplify after adding or subtracting.

$$(a) \frac{x}{x^2+4x-12} - \frac{6}{x^2+4x-12} = \frac{x-6}{(x+6)(x-2)}$$

$$\text{LCD: } (x+6)(x-2)$$

$$(b) \frac{4}{3p} - \frac{5}{2p^2} = \frac{8p-15}{6p^2}$$

$$\text{LCD: } 6p^2$$

$$(c) \frac{6}{5a^2b} - \frac{1}{10ab} = \frac{12-a}{10a^2b}$$

$$\text{LCD: } 10a^2b$$

$$(d) \frac{x-2}{x-6} - \frac{x+2}{6-x} = \frac{x-2}{x-6} - \left(-\frac{x+2}{x-6}\right)$$

$$= \frac{2x}{x-6}$$

$$\text{LCD: } x-6$$

$$(e) \frac{5}{9-x^2} - \frac{4}{x^2+4x+3} = -\frac{5}{x^2-9} - \frac{4}{(x+3)(x+1)}$$

$$\text{LCD: } (x-3)(x+3)(x+1)$$

$$= \frac{-5(x+1) - 4(x-3)}{(x-3)(x+3)(x+1)}$$

$$(f) \frac{x+2}{x^2-36} - \frac{x}{x^2+9x+18} = \frac{x+2}{(x-6)(x+6)} - \frac{x}{(x+6)(x+3)}$$

$$= \frac{(x+2)(x+3) - x(x-6)}{(x-6)(x+6)(x+3)} = \frac{11x+6}{(x-6)(x+6)(x+3)}$$

$$\text{LCD: } (x-6)(x+6)(x+3)$$

$$(g) \frac{t+1}{t+3} - \frac{t-2}{t-3} + \frac{6}{t^2-9}$$

$$= \frac{(t+1)(t-3) - (t-2)(t+3) + 6}{(t-3)(t+3)}$$

$$\text{LCD: } (t-3)(t+3)$$

$$= \frac{-3t+9}{(t-3)(t+3)} = -\frac{3}{t+3}$$

$$\text{LCD: } w-2$$

$$(h) \frac{w+2}{w+2} + \frac{1}{w-2} = \frac{(w+2)(w-2) + 1}{w-2}$$

$$= \frac{w^2-3}{w-2}$$

Simplify each of the following complex fractions.

$$1. \frac{\frac{5x^2}{9y^2}}{\frac{3x}{y^2x}} = \frac{5x^2 \cdot y^2x}{27xy^2} = \frac{5}{27}x^2$$

$$2. \frac{\frac{1-x}{x}}{\frac{1}{1+3x}} = \frac{\frac{1-3x}{x}}{\frac{1+3x}{x}} = \frac{1-3x}{1+3x}$$

$$3. \frac{\frac{6}{2z-10}}{\frac{z-4}{2z^2-8z-10}} = \frac{\frac{6}{1}}{\frac{2z^2-8z-10}{z-4}} = \frac{6(z-4)}{2(z^2-4z-5)} = \frac{3(z-4)}{(z-5)(z+1)}$$

$$4. \frac{\frac{y}{y+3}}{\frac{y}{y+3} + y} = \frac{\frac{y}{y+3}}{\frac{y(y+4)}{y+3}} = \frac{1}{y+4}$$

$$5. \frac{1 - \frac{1}{x} - \frac{6}{x^2}}{1 - \frac{4}{x} + \frac{3}{x^2}} = \frac{\frac{x^2 - x - 6}{x^2}}{\frac{x^2 - 4x + 3}{x^2}} = \frac{x^2 - x - 6}{x^2 - 4x + 3} = \frac{(x-3)(x+2)}{(x-3)(x-1)} = \frac{x+2}{x-1}$$

$$6. \frac{2 - \frac{2}{t+1}}{2 + \frac{2}{t}} = \frac{\frac{2t+2-2}{t+1}}{\frac{2t+2}{t}} = \frac{2t^2}{2(t+1)^2} = \frac{t^2}{(t+1)^2}$$

$$7. \frac{\frac{1}{y+2} + \frac{4}{y-3}}{\frac{2}{y-3} - \frac{7}{y+2}} = \frac{\frac{y-3+4y+8}{(y+2)(y-3)}}{\frac{2y+4-7y+21}{(y-3)(y+2)}} = \frac{5y+5}{-5y+25} = -\frac{5(y+1)}{5(y+5)} = -\frac{y+1}{y+5}$$

$$8. \frac{x^{-2}}{x+3x^{-1}} = \frac{\frac{1}{x^2}}{x+\frac{3}{x}} = \frac{\frac{1}{x^2}}{\frac{x^2+3}{x}} = \frac{x}{x^2(x^2+3)} = \frac{1}{x(x^2+3)}$$

$$9. \frac{x^{-1} + x^{-2}}{5x^{-2}} = \frac{\frac{1}{x} + \frac{1}{x^2}}{\frac{5}{x^2}} = \frac{\frac{x+1}{x^2}}{\frac{5}{x^2}} = \frac{x+1}{5}$$